4734 Probability & Statistics 3

1(i)	$s^2 = 0.00356/80+0.00340/100$ $= 7.85 \times 10^{-5}$	M1 A1 **2**	Sum of variances Or pooled, giving 7.81×10^{-5}
(ii)	$(1.36-1.24) \pm zs$ z=1.96 (0.103, 0.137)	M1 B1 A1 **3**	Must be s, accept t
(iii)	Not necessary since sample sizes are large	B1 **1** **(6)**	Or equivalent. Nothing wrong
2 (i)	Use $\bar{x} \pm z\dfrac{\sigma}{\sqrt{n}}$ $\bar{x} = 337.5/20$ z =2.326 (14.9,18.9)	M1 **B1** B1 A1 4	 3 or 4 SF
(ii)	$1- 0.98^3$ 0.0588	M1 A1 **2**	Use B(3,0.02) or B(3,0.98) for M.
(iii)	Unbiased estimate of σ^2 required t – distribution used to obtain CV	B1 B1 2 **(8)**	
3 (i)	H_0: $p_W = p_N$, H_1: $p_W > p_N$ Pooled $\hat{p} = \dfrac{71+73}{80+90}$ $(=\dfrac{144}{170})$ s^2 = (144/170)(26/170)(1/80+1/90) z = (71/80-73/90)/s =1.381 1.381 < 1.645 Do not reject H_0, there is insufficient evidence that the proportion of on-time Western trains exceeds the proportion of on-time Northern trains	B1 B1 B1 M1 A1 M1 A1 **7**	For both hypotheses. Or π. SR: from $p_1q_1/n_1 + p_2q_2/n_2 = 0.00295$ z = 1.406 B1M1A1M1A1 Max 5/7 If no explicit comparison and correct conclusion then M1A0. Or use P-value or CR In context, not too assertive
(ii)	$s^2 = 71\times9/80^3+73\times17/90^3$ = 0.00295	M1 A1 **2** **(9)**	AEF Allow one error Accept 0.0029
4(i)	Use $L - S_1 - S_2$ μ = 0.7 $\sigma^2 = 0.58^2+ 0.31^2+0.31^2$ = 0.5286 (1-0.7)/σ 0.340	M1 B1 M1 A1 M1 A1 **6**	Or equivalent, or implied May be implied later Correct numerator
(ii)	Use $L - 2S$ with μ=0.7 $\sigma^2 = 0.58^2 + 4(0.31)^2$ - 0.7/ σ - 0.824(5) 0.2048	M*1 B1 Dep*M1 A1 A1 **5** **(11)**	M0 if as (i) unless correct Accept + 0.205 (3SF)

5(i)	Population of differences is normal H_0:$\mu_A = \mu_B$, H_1: $\mu_A < \mu_B$ where μ_A and μ_B denote the population means $\overline{x}_D = 3.222$ $s_D = 5.019$ t = 3.222/(5.019/3) =1.926 CV = 1.860 1.926 > 1.860 Reject H_0 , there is evidence that brand *A* takes less time than brand *B*	B1 B1 B1 M1A1 M1 A1 B1 M1 A1 **10**	*Not "independent"* Or $\mu_D = 0$, $\mu_D > 0$ From formula ,or B2 from calculator Accept 1.93. M1A0 if t = - 1.926
(ii)	One valid reason	B1 **1 (11)**	Data are clearly paired Data not independent
6(i)	37×58/120 17.883.. , 17.88 AG	M1 A1 **2**	Or equivalent
(ii)	H_0: Gender and shade are independent (H_1:--are not independent $3.02^2(14.02^{-1}+14.98^{-1})$ + $6.12^2(17.88^{-1}+19.12^{-1})$ $+3.1^2(26.1^{-1}+27.9^{-1})$ =6.03 EITHER: CV 5.991 6.03 > 5.991, reject H_0 and accept that gender and shade are not independent OR: P(χ^2 > 6.03) =0.049 < 0.05 , reject H_0 and accept that gender and shade are not independent	B1 M1 A1 A1 B1 M1 A1√ **7** B1 M1 A1√	 At least two correct All correct Ft X^2 . Can be assertive. Ft X^2
(iii)	G_1 G_2 G_3 O 29 37 54 E 40 40 40 121/40 + 9/40+196/40 = 8.15 Using df = 2 2.5% tables, 1.7% calculator	 M1 A1 M1 A1 M1 A1 **6 (15)**	 For combining

7(i)	$F(t) = \begin{cases} 0 & t \le 0, \\ t^4 & 0 < t \le 1, \\ 1 & \text{otherwise.} \end{cases}$	B1 B1 **2**	For t^4 For rest
(ii)	$G(h) = P(H \le h)$ $= P(T \ge 1/h^{1/4})$ $= 1 - F((1/h^{1/4})$ $=1 - 1/h$ $g(h) = G'(h)$ $=1/h^2$ $h \ge 1$,(0 otherwise)	M1 A1 A1 A1 M1 A1 **B**1 **7**	Accept < With attempt at differentiation Only from G obtained correctly
(iii)	EITHER: $\int_1^\infty (h^{-2} + 2h^{-3})\mathrm{d}h$ $= \left[-h^{-1} - h^{-2}\right]_1^\infty$ $= 2$ OR: $=1+2\int_1^\infty \frac{1}{h^3}\mathrm{d}h$ $= 1+2\left[-\frac{1}{2h^2}\right]_1^\infty$ $= 2$ OR: $E(1+2T^4)=1+\int_0^1 8t^7\,\mathrm{d}t$ $= 1+[t^8]$ $= 2$	M1 B1 A1 M1 B1 A1 M1 B1 A1 **3** **(12)**	For integrating $(1+2h^{-1})g(x)$, with limits from (ii) Limits not required Limits not required Limits not required